Mathematical mysteries in Byzantium: the transmission of Fermat's last theorem

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When in 1993 Andrew Wiles claimed to have found a general solution to Fermat's last theorem, the announcement indicated a breakthrough in a problem that had fascinated mathematicians for over 350 years.[1] International interest was expressed, since Japanese, French, German and American mathematicians among many others had worked on this problem in number theory, and the Göttingen Academy of Sciences had offered a substantial prize for the first successful solution. Fermat's last theorem claims that 'the equation $x^n + y^n = z^n$ has no non-trivial solutions when n is greater than 2.'[2] And ever since Fermat first propounded it in the Latin notation of the time, and claimed in a famous marginal note: 'I have found a truly marvellous demonstration of this', mathematicians have been trying to confirm what he indicated. The story of their endeavours is now well known, and Wiles's publication, which fills an entire number of *Annals of Mathematics* and draws extensively on Japanese and German theories for particular stages of the solution, has been accepted. He has collected the prize, once a large sum of money but now much reduced by nearly a century of inflation.[3]

Less familiar than the account of this long effort is the history of the text which provoked Fermat's last theorem. The seventeenth-century scholar Pierre de Fermat was a lawyer by profession and an amateur mathematician and physicist. He worked in relative isolation and formulated several original concepts in addition to the celebrated 'last theorem'.[4] One of his sources was the *Arithmetika*, a collection of number problems written by Diophantus, a mathematician who appears to have flourished in Alexandria in the third century AD. This text was famous throughout the Middle Ages and was closely studied by Greeks and Arabs alike. In Byzantium, Diophantus was read, copied and commented on by generations of intellectuals, among them scholars who occupy a strategic position in the transmission of ancient Greek culture to the modern world. His work was translated into Arabic in the early ninth century. It was, however, through the Greek text translated into Latin that Fermat became familiar with the mathematical problems of Diophantus, and in particular the one at Book II, 8, which encouraged the formulation of his own last theorem.

Since its history from Fermat's time onwards is now so well documented, in this article I will examine the means by which Diophantus was preserved for about 1400 years. Why, when so many important works of ancient Greek literature and science disappeared, were these number puzzles preserved? They are not of much practical use, being essentially abstract problems, yet they seem to have met a certain need, or stimulated a curiosity that remained fairly constant from late antiquity to the Renaissance and beyond. Their survival suggests a mathematical and philosophical culture which positively encouraged the text; an articulate awareness of the value of mental calculation, however remote from everyday matters. The medieval context, in which so many technical skills were passed on from generation to generation by word of mouth, may also have favoured an oral tradition of playing with numbers. It is important to remember that there were no Indian numerals at the time; numbers were represented by letters of the Greek alphabet. And since calculations are written out in words, the preserved texts of Diophantus have a very literary look and feel.

The *Arithmetika* originally had thirteen books, of which ten are preserved; Diophantus also wrote a treatise on polygonal numbers, and several other books on mathematics now lost.[5] His major work is an anomaly, rather unusual by Greek standards, being devoted to an investigation of five kinds (ε᾽δη) of numbers that share the same attributes: e.g. squares, cubes, squares of squares, cubes of cubes. He introduced the sign sigma, ς, as a number that shares none of the properties but has an indeterminate multitude of numbers. This is called 'the number', ὁ ἀριθμός. In any problem, ς equals the number whose determination is necessary to the solution.[6]

Within a unified notational system, the problems discussed by Diophantus are quite self-contained; each begins: 'To find …', or 'To divide …' a number or numbers that have specific properties. The techniques were rooted in concrete problems, such as the solution of awkward or difficult divisions of inheritance between a number of heirs, which are documented in Babylonia and Egypt. But by the third century AD these numbering-off techniques had become abstracted so that they could be applied to different problems, and the abstraction had begun to reveal the method lying behind such solutions. In turn the abstract method had become an object of interest. Diophantus did not establish a mathematical investigation of the techniques used in his solutions, nor did he enunciate a developed system of the number theory underlying his procedure. His solutions frequently take off from an intelligent guess, which is worked around with great ingenuity to produce a proof. It has been said that while he matches Euclidean geometry in rigour, Diophantus' proofs fail to convince us of their validity; there is a sense of frustration at their lack of explicit generality.[7] He is usually satisfied with a single answer to his problems rather than a complete solution and it has recently been shown that no general algorithm for Diophantine problems exists. Readers have to go beyond his mathematics to uncover the real workings of these intricate proofs, identified from the Renaissance on as 'the mysteries' of Diophantus.[8]

We know next to nothing of this author, whose skills are recorded in an epigram inscribed on his tomb, composed by an anonymous Greek.[9] It records the phases of his life in an arithmetical riddle which runs as follows: 'God granted him to be a boy for the sixth part of his life, and, adding a twelfth part to this, He clothed his cheeks with down. He lit the light of wedlock after a seventh part, and five years after his marriage He granted him a son. Alas! late-born wretched child; after attaining the measure of half his father's life, chill Fate took him. After consoling his grief by this science of numbers for four years, he ended his life.'

The answer to the puzzle is 84: for he was a boy for fourteen years, a youth for seven, i.e. he became an adult at twenty-one years, at thirty-three he married, at thirty eight he had a son born to him who died at the age of forty two, and he survived him by four years, dying at the age of 84: 38 + 42 + 4 = 84! This is typical of the arithmetical epigrams and riddles so popular in Byzantium.[10] Indeed, the transmission of Diophantus in Greek is associated with a collection of thirty eight of very similar character.[11] Possibly both number problems and riddles were learnt as elements of mental arithmetic, since they are easily memorized and may be preserved by oral tradition. It is clear, however, that in his day, Diophantus was considered very distinguished. Anatolios of Alexandria admired him as 'the most erudite', and when he brought together the most essential parts of mathematical science then known, he dedicated them to his friend Diophantus.[12]

A century or so later, the philosopher and mathematician Hypatia, one of the few female professors of late antiquity, studied the works of Diophantus and Ptolemy, which she expounded in public lectures at Alexandria. She wrote commentaries on both these authors, including at least the first six books of the *Arithmetika*; this may be why they have survived in Greek. Her father, Theon, a famous teacher, had trained her to continue the established education of young men of the day in the traditional style, and from a recent study of Ptolemy's *Almagest*, it is clear that they worked together. Theon deferred to her scholia in his introduction to Book III.[13] Although so little of Hypatia's own writings has survived, her elevated position in Alexandrian society is well documented in the correspondence of Synesius of Cyrene, who greatly admired her. Her commitment to Neoplatonist philosophy involved a life of celibacy and scholarship, which is commemorated in an epigram.[14] Hypatia is better known for the manner of her death than her life: she was the distinguished teacher lynched by irate Christian monks in the great riot of 415, which pitted supporters of Patriarch Cyril against the Prefect Orestes and other non-Christian groups.[15]

Yet the riot did not quench enthusiasm for Diophantus who continued to be studied in Christian Byzantium. Testimonia of his works have been mapped by Tannery, and include a curious reference in the *Life* of St John of Damascus, who is said to have studied Diophantus, together with his adopted brother Kosmas the Hymnographer, in late seventh-century Palestine.[16] As Cyril Mango has shown, however, the late antique traditions of a basic education, including the quadrivium of mathematics, appear to have lived on longer in Palestine than in other parts of the Byzantine world.[17] The suggestion that young men in the 680s and 690s, destined for ecclesiastical careers, could have studied Pythagoras, Diophantus, Euclid and music is remarkable. Their own writings reveal a solid background knowledge of ancient scholarship, for example Kosmas' commentary on the mythical elements in Gregory of Nyssa. Palestinian monasteries sustained a developed educational system, probably better than that offered in the Byzantine capital, Constantinople. For as we shall see, after the middle of the seventh century, higher education and, specifically, training in advanced mathematics seems to have been hard to find.

In his recent study of Diophantus, André Allard has clarified the medieval transmission of the Greek text, positing an archetype now lost that may fill the same role as intermediaries hypothesized by previous editors.[18] From this text two families descend, both represented by manuscripts of the thirteenth century. One is based on a copy from which a manuscript now in Madrid was made;[19] the other consists of a copy made by Maximus Planudes himself, who worked on the text in the 1290s. Only a few folios of his autograph are now preserved in Milan.[20] The first version was known to the famous Renaissance teacher of Greek, Constantine Laskaris, who left his copy to the Cathedral Chapter of Messina, the Sicilian city in which he lived and taught.[21] From there it passed to Madrid like so many important Byzantine manuscripts now preserved in Spain, such as the illustrated Skylitzes, and the Middle Byzantine Taktikon of 971-5 in the Escorial. This library also preserves a copy of Michael Psellus' letters; in a long one on mathematics, he mentions his own copy of the *Arithmetika* of Diophantus.[22] The second version, with the Planudean commentaries is represented by two important texts in the Vatican library, to which I will return.

In parallel with the medieval Greek transmission, the Arabs were also busy studying Diophantus, whose writings they translated.[23] From a recent discovery in the Isfahan library, it has been established that they had access to a fuller version of the *Arithmetika*.[24] Four previously unknown books must have been placed between Books III and IV as preserved in the Greek tradition. The Arabic translation appears to have been made in the early ninth century, during the rule of Caliph al-Mamun (813-33), which was a particularly important period in the development of this branch of mathematics. It is at this moment that Arab scholars coined the term 'algebra', which was used to describe the theorems of Diophantus and others. It occurs for the first time in the title of al-Khwarizmi's *Book of the Calculation of the Algebra and the Al-muqabala*, written under the patronage of al-Mamun.[25] The popularity of Diophantine theorems in the tenth and eleventh centuries is evident from the quarrels that arose between three philosophers, al-Khujandi, al-Khazin and Abu Ga'far, who were all trying to demonstrate a proof of one theorem to the third power.[26]

For both Arabs and Greeks the appeal of Diophantus seems to have derived from his number theory, which often coincided with the medieval fascination with number magic. It may also be connected with the popularity of epigrams and collections of arithmetical riddles and puzzles, much beloved by peoples ancient and medieval, Christian and Muslim.[27] In the early seventh century when the Armenian scholar, Ananias of Shirak, was searching for instruction in advanced mathematics, he may well have used a Greek collection in the compilation of his Armenian one, devised 'for retelling at feasts'.[28] The fact that he includes riddles with contemporary references to Constantinople, for instance in the puzzle to calculate the number of pounds of gold distributed to the clergy of the cathedral church of St Sophia, suggests familiarity with topical problems transmitted in Greek.[29] Not only are these puzzles curious brain-teasers; they can easily be memorized and transmitted orally with no reference to a written text. This is obviously an important aspect of the long history of recreational mathematics.

From the Caliphate of Baghdad, Muslim scholars translated and mastered ancient Greek science, making their own very significant contributions to mathematics, as if in competition with the medieval Greeks. The process may be illustrated by a story, probably apocryphal, recorded in the ninth century during the reigns of Caliph al-Mamun and the Emperor Theophilos, when a Byzantine student was taken prisoner by the Arabs and found himself in a Baghdad jail.[30] To entertain his fellow prisoners he expounded theorems and number puzzles. When this fact was observed, he was summoned to the Caliph who asked him to demonstrate his knowledge. It transpired that the Arab mathematicians had translated the works of Euclid, but could not prove his theorems. The Greek prisoner was ordered back to Byzantium to persuade his teacher to visit Baghdad. But when Theophilos heard of the proposal, he instead appointed the said teacher to give lessons at the church of the Forty Martyrs in Constantinople.

This expert was Leo the Mathematician, also called the Philosopher, whose life and education remain rather obscure, since the teacher from whom he acquired most of his knowledge is described as 'a wise man' who lived on Andros.[31] In the mountains of the same Aegean island, Leo found books which he studied and took with him when he returned to the capital. According to Theophanes Continuatus, Leo followed the ancient principle of seeking a learned man and studying with him, in order to overcome the lack of advanced training in mathematics and philosophy in Constantinople. Having learned all that his teacher could impart, he then went off in search of manuscripts, and eventually set up as a private teacher in Constantinople.[32]

Once the emperor realized what an important scholar Leo was, paid employment followed in a school established at the church of the Forty Martyrs. There is no suggestion that this position involved an official church post, but Leo was soon ordained, for in 840 Theophilos appointed him to the archbishopric of Thessalonica, a very senior ecclesiastical position. There he remained for three years while the iconoclast policies of the emperor were in force. If we may conclude that Leo was considered a reliable opponent of the iconophile veneration of images, he would have been a colleague of the iconoclast Patriarch, John the Grammarian, also a scholar of advanced skills.[33] After the restoration of the images, he must have left his ecclesiastical position and seems to have returned to teaching. In 863, the acting head of government, Bardas, made him head of the Magnaura school, where he held the title 'chief of the philosophers' and taught both philosophy and the quadrivium of mathematics, assisted by Theodore (his pupil) who taught geometry, Theodegios, for astronomy, and Kometas, for grammar. Arethas, a younger scholar and bibliophile, heard Leo lecture on Euclid, and incorporated Leo's commentary into his commissioned copy of the *Elements*.

Leo's importance in the preservation of ancient Greek mathematics rests on his role in the production of manuscripts that form a vital link in the line of descent from antiquity. Because this can be demonstrated in the case of several key texts, it seems quite reasonable to associate him with the transmission of Diophantus. But first let us survey the surviving manuscripts, notably the copy of Euclid's *Elements*, made by Stephanos, *klêrikos* in AD 888, which was commissioned by Arethas.[34] It contains Leo's *hypomnêma scholikon*, a commentary in the form of a scholion, on definition 5 of Book VI, which Arethas had heard expounded by Leo.[35] A very early minuscule text of Ptolemy, *Syntaxis*, dated to the ninth century is also preserved, Vaticanus graecus 1594, which contains a marginal note: 'the book of Leo, the excellent astronomer'.[36] While the note has been shown to be a much later addition, there is no reason to dissociate this manuscript from Leo and his circle. A similar tribute closes a version of the works of Archimedes. Two later copies preserve the invocation: 'May you prosper, Leo the geometer, may you live many years, much the dearest to the Muses.'[37] From this it seems likely that Leo was responsible for having the text of Archimedes copied.

In connection with the transmission of Archimedes, an additional aspect of Leo's importance derives from the fact that many of the manuscripts had to be transcribed from uncial into the novel minuscule script of the ninth century.

This new cursive script was developed during the theological controversy over icons and facilitated quicker writing. But for the copying of ancient texts, a complete process of transliteration was required, and it seems clear that Leo took a major role in organizing the method of conversion. The dangers inherent in this activity may be documented by the famous codex of Giorgio Valla (c. 1430-99), now in Paris, which has a long colophon describing the transcription, 'made from a very old ... exemplar, which exhibited a very great, even immeasurable lack of clarity because of mistakes'. Paul Lemerle suggested that some of these errors were the result of copying a text in uncial script into minuscule during the period of transliteration from one letter form to the other, which took place under Leo's direction.[38] Leo's scholarship also forms the basis of the incomplete Archimedes compiled by Isidore, which formed the archetype (lost in the sixteenth century) for a twelfth-century copy that passed through the Norman kings' library in Sicily and on to the Vatican. There it was used by William of Moerbeke in his translation of Archimedes into Latin.

Given his position as the leading philosopher and mathematician in ninth-century Byzantium, Leo should probably be connected with the lost version of Diophantus posited by Tannery and Heath.[39] The suggestion that the *Arithmetika* must have been copied in the eighth or ninth century and that this copy served as the model for the thirteenth-century version preserved in Madrid ignores the dearth of evidence for any mathematical innovation in the eighth. In the ninth, however, when renewed interest generated by Leo was encouraged by the schools set up by Theophilos and Bardas, an obvious context emerges. Leo's interests may also be reflected in copies of Apollonius *On Conics*, which served as the prototype for two copies made in the tenth and twelfth centuries, both now in the Vatican library, and of Theon of Alexandria on astronomy, bound with a treatise on geometry by Proclus of Xanthos, and a treatise on mechanics by Kyrinos and Markellos. The combination of this copying and transliterating activity with Leo's teaching also generated manuscripts on astronomy, a short treatise, and further interpretations of Archimedes.

In addition, Leo is associated with several literary manuscripts: he owned copies of Porphyry, for which he composed a distich (*AP*, IX, 214) and Achilles Tatius' *Leucippe and Clitophon*, on which he probably composed another (*AP*, IX, 203). This literary interest is the link to another of Leo's activities, namely the writing and arranging of epigrams in a *sylloge* (collection). He not only produced his own, autobiographical verse full of Homeric allusions, in which he identifies himself as a Hellene (*AP*, XV, 12); he also made contributions to Book IX of the *Greek Anthology*. Many of these epigrams are transmitted in series which suggests that they were taken from a late-antique anthology possibly by Leo himself.[40]

Writing epigrams was an activity shared by Leo's contemporaries, including his student and rival, Constantine the Sicilian, who contributed to Book XV of the *Greek Anthology*. In an important manuscript, the Barberinianus graecus 310, Leo, Constantine, and Theophanes (another contemporary) are also bracketed together as ninth-century *grammatikoi*, who all wrote anacreontics. At a gap in Book XV, the scribe J brings the three together, referring to Leo as the Hellene and the other two as *makarioi* (recently deceased). This occurs in his *sylloge* of epigrams more contemporary than ancient, including some by Michael the Chartophylax, who played a major role in the transmission of the *Anthology*.[41]

Given Leo's mathematical interests, might he also have copied the arithmetical epigrams that form a large part of Book XIV of the *Greek Anthology*? This collection goes back to Metrodoros, an author who flourished at the turn of the sixth century, and provides epigrams 116 to 146 which are very close to the numbering-off tradition of Diophantus. The same book also contains the thirty-eight found in Diophantus' manuscript, beginning with the epigram attributed to Socrates, *AP*, XIV, 1, which follows the established style of a mathematical riddle or number game.

As an example of this type of epigram, I cite XIV, 3:

> Cypris thus addressed Love who was looking downcast: 'How, my child, hath sorrow fallen upon thee?' And he answered: 'The Muses stole and divided among themselves, in different proportions, the apples I was bringing from Helicon, snatching them from my bosom. Clio got the fifth part, and Euterpe the twelfth, but divine Thalia the eighth. Melpomene carried off the twentieth part, and Terpsichore the fourth and Erato the seventh. Polymnia robbed me of thirty apples and Urania of a hundred and twenty, and Calliope went off with a load of three hundred apples. So I came to thee with lighter hands, bringing these fifty apples that the goddesses left me.'

Solution: 3360.[42]

The same problem is addressed in several more devoted to apples (XIV, 117-199), while 120 is a division of walnuts from a tree (cf. 138). The epitaph of Diophantus himself is preserved here (126); an identical structure describes a certain Demochares (127); and 139 is a mathematical problem about the measurement of time, based on a sun-dial addressed to Diodorus, the great glory of dial-makers. The combination of arithmetical epigrams and the epitaph of Diophantus in this form perhaps points to another slight connection between Leo and the author of the *Arithmetika*.

Leo's name is also attached, incorrectly, to a fragment of an epic poem on the Arethusa spring in Sicily, dating from the Roman period (IX, 579). Nestor of Laranda may be the author of this whole series devoted to rivers, springs, sources and so on, and the attribution to Leo derives from his authorship of the preceding epigram, 578. Similarly, other epigrams are attributed to Leo (e.g. IX, 361), although in the *Sylloge Euphemiana*, dating from the reign of Leo VI (886-913), the poem is accurately described as anonymous. Such attributions reflect Leo's fame as a composer of verses as well as a mathematician and philosopher.

I have emphasized Leo's contribution to the development of mathematical as well as literary traditions in Byzantium not only because he seems the person most likely to have commissioned a ninth-century copy of Diophantus, but also because he embodies the wide range of skills associated with the study of philosophy in medieval times. All the works of the ancients were important to these scholars, who made no distinction between the need to use metre correctly and the need to work out scientific problems accurately. To the Byzantines Leo was famous as both a mathematician and a philosopher. He has justly been called 'the first example in Byzantium of a truly "Renaissance man"',[43] a polymath with a passion for all things ancient. His heritage was not lost in Byzantium, although it is difficult to trace the study of Diophantus through the tenth century. Many of the gaps in the written record may be filled by oral transmission. The compilers of the *Suda* knew of the importance of Diophantus in the tradition of number theory and recorded the commentaries made to the first six books by Hypatia (quoting from the *Life* of Isidore by Damascius).[44] In 1007-8 the *Quadrivium* of higher learning was revised in Byzantium, indicating persistent concern to ensure the study of the 'mathematical' quartet, although Diophantine equations did not figure in it. In the mid-eleventh century, however, Michael Psellus owned a copy of the *Arithmetika* and wrote a long letter about mathematics.

Between this mention of Diophantus and the intensive study of him by the late Byzantine authors George Pachymeres and Maximus Planudes there is another breach in our knowledge, although we know that progress was made in mathematics, including the introduction of Indian (Arabic) numerals and Indian (Oriental) methods of calculation. Towards the end of the thirteenth century Pachymeres made a partial paraphrase[45] of Diophantus' *Arithmetika* and incorporated it into his introduction to the *Quadrivium*,[46] while Planudes produced a systematic commentary on the first book and part of the second.[47] According to Nigel Wilson he 'manages to set out some of the problems in a form which is easier to understand than that of the original and not much harder than that which they might assume in modern notation'.[48] At II, 8, there is the remarkable scholion: 'May your soul, o Diophantus, rest with Satan on account of the difficulty of your other theorems and particularly of the present theorem', which is perhaps a consequence of this editorial work of Planudes. So the mysteries of Diophantus were still unyielding. But the same scholion in the Madrid manuscript has been authoritatively attributed by Wilson to a later hand, that of the polymath John Chortasmenus.

Nonetheless, to reach this level of mastery over the text, Maximus Planudes took pains to collate his own copy of Diophantus against another which he wanted to borrow from Manuel Bryennius (letter 33), and yet another which he lent to Theodore Mouzalon, the grand logothete.[49] Planudes' autograph of the edition and commentary of Diophantus, written in 1292-3, survives in only ten folios now in Milan.[50] But it is only one of the many manuscripts produced at the Chora monastery in Constantinople, where Planudes had access to an excellent library. There he also devoted expert attention to ancient geographers, studying both Ptolemy and Strabo: his enthusiasm at finding a text of Ptolemy is celebrated in hexameter verses which accompany his careful edition of 1295-6 (Vaticanus graecus 177).[51] A later copy of Ptolemy is found in the same manuscript as Diophantus, together with an astronomical work by Theodosius, confirming Planudes' wide range of scientific interests. His autograph corrections to parts of Strabo's geography are preserved in a Paris text (Parisinus graecus 1393), which includes later classical authors, such as Pausanias. These justly celebrated manuscripts demonstrate the erudition of late Byzantine humanists including Nikolaos Artabasdos Rhabdas and Demetrios Kydones, who mentions his own study of Diophantus and Euclidean geometry in letter 347.[52] There is no sense of division between literary and scientific subjects in their enthusiasm. Rather, they cherished the wisdom of the ancients and sought by all possible means to preserve and perpetuate it.

Thus Maximus Planudes was also responsible for reorganizing and editing the *Greek Anthology* from a fuller version than that preserved in surviving manuscripts. From this he was able to restore 388 epigrams missing from the second half of Book IX, which today forms Book XVI, the so-called Planudean appendix. While none of them take the form of arithmetical riddles, Planudes rescued a vast number of verses dedicated to statues, as well as reproducing epigrams from the monuments in the Hippodrome of Constantinople.[53] In this activity, he makes the same connection between the epigram and arithmetic as Leo before him. But when it came to ancient culture, his real love was reserved for Plutarch, whose *Moralia* he copied in his own hand, because he greatly liked the man.[54] As a monk of the Chora monastery, Planudes bears witness not only to its resources for research but also to the far-ranging intellectual curiosity of late Byzantine scholars.

This tradition was maintained throughout the fourteenth and fifteenth centuries, influencing those Byzantines who emigrated to the West to teach Greek and promote the study of ancient texts. Within the Constantinopolitan circle John Chortasmenus played a distinguished role. A product of the patriarchal chancellery in Constantinople, he wrote iambic verses and epigrams, a *Life* of Constantine and Helena, prolegomena to the *Logic* of Aristotle, orations and numerous letters, among other works. His collection of manuscripts reveals a very deliberate bibliophile, who copied astronomical texts for his own use and was responsible for the rebinding of the famous sixth-century herbal by Dioscorides dedicated to Juliana Anicia.[55] He too studied Diophantus and is now recognized as the author of the despairing scholion in the margin at Book II, 8.[56] As the teacher of Bessarion he probably had an impact on the young scholar from Trebizond, who in turn became an avid collector of Greek manuscripts. Certainly he imparted to the man whose library became so famous a sense of the inter-connected relationship between mathematical, philosophical and literary studies, which had characterized Byzantine scholarship since the teaching of Hypatia in late antiquity. Bessarion, as is well known, followed an ecclesiastical career, converted to western catholicism and was made a cardinal in 1439. But he wrote prolifically on many topics, not only theological, and his interest in scientific and philosophical works is amply demonstrated by his collection, which was bequeathed to the Republic of Venice.[57]

Among these manuscripts is one important copy of Planudes' edition of Diophantus, which was to play a crucial role in the transition from East to West. It was brought to Italy by Bessarion and is now codex 308 of the Marciana library.[58] In 1464 the Renaissance scholar, Regiomontanus, who served as secretary to the Cardinal, knew that it contained only six books, not the promised thirteen, but reported: 'it is really most wonderful and most difficult … I should like to translate it into Latin, for the knowledge of Greek which I have acquired while staying with my most reverend master Bessarion would suffice for this'.[59] At about this time, 1463-4, in a public lecture, an *Oratio* given at Padua, he observed: 'No one has yet translated from the Greek into Latin the fine thirteen Books of Diophantus, in which the very flower of the whole of Arithmetic lies hid, the *ars rei et census* [the art of the thing and its riches] which today they call by the Arabic name of "Algebra"'.[60] While this 'hidden' character was perhaps enhanced by the riddle-like form of transmission, its very difficulty and obscurity served to confirm the mysterious quality of Diophantus' work.

Thus, only a decade after the fall of Constantinople to the Turks, one manuscript of Diophantus had been identified and one western scholar was anxious to translate it into Latin. Another copy was already recorded in the Vatican catalogue of Greek manuscripts, Vaticanus graecus 304, which entered the papal library with Pope Nicholas V (1447-55) before 1453.[61] It lacks the Planudean commentaries, so must represent the Madrid family, and also contains the Tables of Theon of Alexandria. Two other copies of Diophantus came to the Vatican in the course of the sixteenth century, one made from Bessarion's Venice manuscript.[62]

But no use was made of this work until the mid-sixteenth century, when algebra became a new classical subject. Among scholars responsible for this endeavour, Francisco Maurolico, the son of Greek refugees from Constantinople, spread an interest in number theory through his study of arithmetic in two books published in 1575 after his death.[63] More influential was the work of Raphael Bombelli who attempted to theorize the discipline using the *Arithmetika* of Diophantus, which he intended to translate. Together with a colleague, he prepared a Latin version of several books, but this was never published.[64] Instead, Bombelli incorporated all the first four books and some of the fifth (148 problems in all) into his own *Algebra* (published in 1572) without distinguishing the reasoning of Diophantus from his own. Thus Diophantus got no credit and his text remained unknown.

At about the same time, however, more useful contributions to the transmission of Diophantus were made by professional scribes, who copied his work for patrons. The most important of these is the famous Giovanni of Otranto, who signed his copies using the Latin form, Ioannes Honorius Hydruntinus[65] From 1535-56 he served as Greek scribe and restorer of manuscripts in the Vatican Library, and was responsible for copying a large number of scientific works, including Diophantus. Realizing that in two Vatican manuscripts he had different traditions of the *Arithmetika*, he collated Vaticanus graecus 304 of the Madrid family, and Vaticanus graecus 200, a copy of Bessarion's copy of Planudes' edition, to form a composite more complete than either. This piece of editorial sophistication combined the earlier tradition represented by Madrid with the late thirteenth-century edition and commentary. Giovanni was a prolific copier; in some manuscripts he used his daughter as an assistant; and together they produced many texts of mathematics, the *Conics* of Apollonius, Nicomachus, Ptolemy, Euclid, and a large number of early Christian writings. His Diophantus was taken to France where in the 1570s François Viète read it (Parisinus graecus 2379).[66]

By a similar process of sensitive editing, Andreas Dudicius Sbardellatus, a scholar active in Poland, commissioned a Venetian scribe to make a composite copy of two later versions (Ambrosianus A91 and Guelferbytanus Gudianus 1) which is now Reginensis 128.[67] This is the text from which the first Latin translation of Diophantus was made in 1571-5 by Wilhelm Holzmann, who called himself Xylander, 'wood man', in Greek.[68] It had been carefully prepared, yet Xylander complained of enormous difficulties in the very corrupt text. More mysteries, then, which may have been especially discomforting to a scholar who prided himself on his skill at editing Plutarch, Stephanus and Strabo. His Latin translation was published in 1575 and dedicated to Prince Ludwig of Wittenberg: the mysteries of Diophantus were beginning to yield.

Xylander also planned to publish the Greek text, and his edition is preserved, apparently prepared for printing, in another manuscript, Palatinus graecus 391. But the first Greek edition only appeared much later, in 1621; it was made by the French scholar Bachet de Méziriac using Parisinus graecus 2379, the composite Greek copy put together by Giovanni of Otranto.[69] Bachet included the Latin translation of Xylander, corrected and improved in ways that were intended to enhance his own contribution and downplay that of Xylander. He also consulted Bombelli's translation which he found better than Xylander's in some cases.

It was this same edition of the Greek that was re-published with Fermat's notes by his son in 1670.[70] The notes had been taken from Fermat's copy of Bachet's edition of Diophantus, now lost, in the margins of which he had noted at Book II, 8: 'On the other hand it is impossible to separate a cube into two cubes or a biquadrate into two biquadrates, or generally any power except a square into two powers with the same exponent. I have discovered a truly marvellous proof of this, which however the margin is not large enough to contain'.[71] He thus formulated the theorem that became known as Fermat's last theorem and which Andrew Wiles has now solved. Whether Fermat thought he had a general proof, or a proof that the theorem was not correct to the third and fourth power, is not entirely clear. But ever since his claim, mathematicians have been trying to demonstrate that the proposition has no non-trivial solutions. In 1753 Euler reported to Goldbach that he had proved Fermat's theorem to the third and fourth powers, but could not find a method to prove the fifth, which seemed to require a different technique.[72] That was achieved by Legendre and Dirichlet before 1825, yet any hope of finding a general solution seemed remote at the beginning of the twentieth century, when Paul Wolfskehl established a very handsome prize at the Göttingen Academy of Sciences.[73]

The study of Diophantus in modern times is now well-known, and brilliantly chronicled in Simon Singh's recent study. However, Fermat would never have learned of the problem but for the patient work of scholars who for over a millennium kept these mathematical puzzles alive. And it was in Byzantium, a part of the medieval world that cherished the memory of all things Greek, that the mysteries of Diophantus were preserved, embellished, developed and enjoyed by many generations of amateur mathematicians like Fermat. Much less evidence and detail of their work survives than the documentation for problem-solving in the Renaissance and later eras. But I hope that this article has shown how crucial their concern was for the modern understanding of ancient Greek mathematics.

It used to be said that the fall of Constantinople to the Turks provided an important stimulus to the Renaissance.[74] But this claim has been so much debunked that it has been rendered impotent. So there is a certain satisfaction in observing that the transmission of Diophantus to the modern world in its original language occurred through the medium of a manuscript brought to Italy by the infamous Cardinal Bessarion, an apostate from Byzantine Orthodoxy. Through his collection of Greek manuscripts, given to Venice, the West acquired what became the core of the Marciana library. Among them, his own manuscript of Diophantus was in turn copied several times; collated with an earlier version by a particularly intelligent scribe, Giovanni of Otranto, in the mid-sixteenth century; and edited in Paris by a humanist scholar Bachet de Méziriac, with input from Holzmann (Xylander) who made the first Latin translation. There is, thus, a direct link back to Byzantium where intellectuals, such as Leo the Mathematician (a humanist before his time), and monks such as Maximus Planudes, not only copied and studied the mysteries of Diophantus, but also wrote commentaries that tried to make sense of them. Their work highlights a continuous thread of fascination with ancient mathematics otherwise represented by orally preserved riddles and epigrams of number theory passed on from generation to generation in ways that remain largely hidden.

The association of mathematical riddles with collections of epigrams, such as those in the *Greek Anthology*, is common to many cultures. Recreational brain teasers were popular not only in medieval Byzantium, but also in ancient China, India, Egypt and even in seventeenth-century France, where Bachet the editor of Diophantus, made his own collection.[75] Here is an example: 'If you multiply 5525, a number composed of six squares … into 1073, a number composed twice … the produce is 5929325, a number composed (and this is marvellous) twenty-four times from two squares' – and he gives the sides. This is precisely the type of mental exercise that the Byzantines used to enjoy, a game with numbers that can be transmitted without writing. Because of the similarity between such problems and the equations of Diophantus, we can image the oral channels of communication that kept his number theories alive between late antiquity and the Renaissance. As for the written forms, it is perhaps the abiding curiosity about the properties of numbers that accounts for the perseverance with which generations of Byzantine scholars struggled with the mysteries of Diophantus.

NOTES

1 I am most grateful to Professor Wiles for conversations about this paper and about the way in which mathematicians have evaluated proof over the centuries. For coverage of the discovery, see G. Kolata, 'Andrew Wiles: A Math Whiz Battles 350-Year-Old Puzzle', in *Math Horizons* (Winter 1993) 8-11. I particularly thank George Baloglou for sending me a copy of this item. The recent publication by S. Singh, *Fermat's Last Theorem* (London 1997) gives copious details.
2 Kolata, 'Wiles', 8; A. Baker, *A Concise Introduction to the Theory of Numbers* (Cambridge 1984) 200.
3 Singh, *Fermat,* 116, 135-7, 143-6, with extraordinary detail on the prize money established by Paul Wolfskehl and administered by the Göttingen Academy of Sciences. In the course of investigating its history I have been introduced to a whole new world of recreational mathematics and complex number riddles, which would have remained unexplored but for the coincidence of the solution announced by Wiles and the interest of colleagues at Princeton University associated with the Program in the History of Science, and the Program in Hellenic Studies. Without their help and guidance it would have been impossible for me to pursue the mysteries of Diophantus through the Byzantine era. I would also like to acknowledge the assistance of many friends and students for questioning my presentations and improving my understanding of the issues: Meg Alexiou and Greg Nagy, who provided the first stimulus by inviting me to a Harvard seminar entitled *Reading Antiquity Backwards* in May 1995; Dominic Rathbone and members of the KCL graduate seminar; Robert Thompson and members of the Oxford Byzantine Seminar; the late Simon Pembroke and an ICS seminar; and Pat Easterling and Michael Reeve and members of a Cambridge manuscript group. To Nigel Wilson, Giles Constable and Anthony Barnett my warmest thanks for critical advice.
4 On Pierre de Fermat, I am greatly indebted to Mike Mahony, whose study, *Pierre de Fermat* (new ed., Princeton 1993) has been a revelation of the culture of seventeenth-century mathematics. On his other theorems and concepts, such as Fermat primes, an optimization theory (Steiner's problem) and the principle of least time (physics), see also Baker, *Introduction*; Singh, *Fermat*, 37-47, 60-72.
5 Diophantus is an exceptionally difficult author of algebraic problems: see N.G. Wilson, *Scholars of Byzantium,* rev. edn. (London 1996) 42. On Diophantus' contribution to the history of mathematics, see T.L. Heath, *A History of Greek Mathematics*, 2 vols. (Oxford 1921), II, 448-517; and on his significance in Byzantium, A. P. Kazhdan et al. *Oxford Dictionary of Byzantium* (3 vols., Oxford 1991), II, 1313-14.
6 For all that follows on Diophantus and the way his work was read in the Renaissance, see the outstanding study by J. S. Morse, 'The Reception of Diophantus' arithmetic in the Renaissance', Diss. Princeton, 1981; also T. H. Heath, *Diophantus of Alexandria. A Study of the History of Greek Algebra* (2nd ed., Cambridge 1910) 32-7.
7 For explication of Diophantus' procedure, I am much indebted to discussions with Michael Mahony; see also the thesis by Morse, which was written under his supervision, 'Reception', 9-11.
8 Morse, 'Reception', 15.
9 *The Greek Anthology*, XIV, 126, tr. W. R. Paton (Cambridge, Mass. 1979), 5. 92-4; see also Heath, *History*, II, 441-3.
10 The next one in the same collection of Metrodoros concerns Demochares and follows exactly the same pattern, XIV, 127; Paton, 5. 94.
11 See the critical edition of Diophantus by P. Tannery, *Diophanti Alexandrini Opera omnia*, 2 vols. (Leipzig 1895), II, 43-72; the discussion of Metrodoros in Heath, *History,* II, 442-3; Alan Cameron, 'Michael Psellus and the date of the Palatine Anthology', *Greek, Roman and Byzantine Studies*, 11 (1960) 339-50.

12 See K. Vogel in *The Dictionary of Scientific Biography* (New York 1971), IV. 110-19.
13 Alan Cameron, 'Isidore of Miletus and Hypatia on the editing of mathematical texts', *Greek, Roman and Byzantine Studies* 31 (1990) 103-27; M. Dzielska, *Hypatia of Alexandria* (Eng. edn. Cambridge, Mass. 1996) 70-72; M. E. Waithe (ed.), *A History of Women Philosophers* (Dortrecht 1987), I, 175, notes that Tannery spotted Hypatia's use of the sexagesimal fraction in working out solutions, which was also observed by Rome in his edition of the commentary on Book III of Ptolemy's *Almagest*, see A. Rome, *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste,* vol. III (Studi e Testi, 106, Rome 1943), cxvi-cxxi. Waithe suggests that this seems to be particular to her and may identify her own contribution, I, 176. For a more detailed analysis of her method, with clear translations and commentary, see now W. R. Knorr, *Textual Studies in Ancient and Medieval Geometry* (Boston 1989) 753-804.
14 *AP* IX. 400, which is not about a Christian virgin, *pace* Alan Cameron, *The Greek Anthology from Meleager to Planudes* (Oxford 1993) 323-5. See M. Lauxtermann, 'The Byzantine epigram in the ninth and tenth centuries', Diss. Amsterdam 1994, 243, n. 131. See also the testimony of Synesius on Hypatia's brilliance as a teacher analysed by Dzielska, *Hypatia*, 28-55, 58-65.
15 On the death of Hypatia, see Socrates, *Ecclesiastical History*, VII, 13-15; Sarolta Takács, 'The magic of Isis replaced', *Poikila Byzantina* 13 (Bonn 1994) 491-507; J. Rougé, 'La politique de Cyrille d'Alexandrie et le meurtre d'Hypatie', *Cristianesimo nella storia* 11 (1990) 485-504.
16 This would be the first mention in medieval times, but since it was recorded by a twelfth-century Patriarch of Jerusalem, many centuries after the event, it may not be altogether reliable. B. Hemmerdinger, 'La Vita arabe de saint Jean Damascène et BHG884', *Orientalia Christiana Periodica* 28 (1962) 422-3, has shown that this Greek version is based on an Arab original and must have been copied between 808 and 969, when Patriarch John died.
17 C. Mango, 'Greek culture in Palestine after the Arab Conquest', in G. Cavallo et al., eds. *Scritture, libri e testi nella aree provinciali de Bisanzio* (Spoleto 1991) 149-60. Another allusion to Diophantus occurs in Ignatius the Deacon's *Life* of Tarasius, Patriarch of Constantinople from 784 to 806. There is, however, no suggestion that either Tarasius or Ignatius studied his theorems: S. Efthymiades, *The Life of Patriarch Tarasios by Ignatios the Deacon* (Aldershot 1998) 95.
18 A. Allard, 'La tradition du texte grec des Arithmétiques de Diophante d'Alexandrie', *Revue d'Histoire des Textes* 12/13 (1982/3) 57-137; and, 'Les Scholies aux Arithmétiques de Diophante d'Alexandrie dans le Matritensis Bib. Nat. 4678, et les Vaticani gr. 191 et 304', *Byzantion* 53 (1983) 664-760. Cf. Heath, *Diophantus of Alexandria,* 14-15. Following the link posited by P. Tannery, *Diophanti*, II, xviii, Heath suggested that this important copy must have been made in the eighth or ninth century.
19 This is Matritensis Bib. Nat. 4678 (previously 48), see G. de André, *Catalogo de los Códices Griegos de la Biblioteca Nacional* (Madrid 1987), cos. 128, 227-8. It entered the library in 1712.
20 Allard, 'L'*Ambrosianus Et. 157 Sup.*, un manuscrit autographe de Maxime Planude', *Scriptorium* 33 (1979) 219-34; see also Wilson, *Scholars of Byzantium*, 233.
21 See Morse, 'Reception', 189-91; Allard, 'Les Scholies', 667.
22 See Tannery, *Diophanti*, II, 37-42.
23 Heath, 19, citing Ibn al-Nadim, author of *The Fihrist* (tr. B. Dodge, New York 1970); cf. J. Stroyls, 'Survey of the Arab contributions to the theory of numbers', in *Proceedings of the First International Symposium for the History of Arab Science*, vol. 2 (Aleppo 1978) 173-9.
24 See R. Rashed, 'Les travaux perdus de Diophante', *Revue d'Histoire des Sciences* 27 (1974) 97-122; and 28 (1975) 3-30. The same author has now published an Arabic translation of the four previously unknown books from a mid-ninth century translation of Diophantus' work (Cairo 1980); cf. J. Sesiano, *Books IV to VII of Diophantus' Arithmetica in the Arabic Translation Attributed to Qusta ibn Luqa* (New York 1982). The recent Budé edition, also by R. Rashed, provides a French

translation, see Diophante, *Les Arithmétiques*, vols. 3 and 4 (Paris 1984). They must have been placed between Books III and IV in the original Greek.

25 On al-Khwarizmi, see R. Rashed, 'L'idée de l'algèbre chez al-Khwarizmi', and 'La nouvelle analyze diophantienne: l'exemple d'al-Khazin', both reprinted in *Entre Arithmétique et Algèbre* (Paris 1984) 17-29, 195-225.

26 Rashed, 'Nouvelle analyse', 220-25.

27 On the Chinese and Indian contributions to recreational mathematics, in which medieval Christians and Arabs also delighted, see H. Hermelink, 'Arab recreational mathematics as a mirror of age-old cultural relations between Eastern and Western civilizations', in *Proceedings of the First International Symposium for the History of Arab Science*, vol. 2 (Aleppo 1978) 44-52; J. Sesiano, 'Arabische Mathematik im 8.-10. Jh.', in P.L. Butzer and D. Lohrmann (eds.), *Science in Western and Eastern Civilization in Carolingian Times* (Basel 1993) 399-442.

28 K. Vogel, 'Byzantine Science', in *Cambridge Medieval History*, rev. edn., IV. II, (Cambridge 1966) 268. Ananias' 24 riddles in Armenian have been translated into Russian, by I. Orbeli (Petrograd 1918), and into German, by P.S. Kokian, 'Des Anania von Schirak arithmetische Aufgaben', *Zeitschrift für die Österreichischen Gymnasien* 69 (1919-20) 112-17. Two examples are published in French translation by J.P. Mahé, 'Quadrivium et Cursus d'Etudes au VIIe siècle en Arménie et dans le monde byzantin', *Travaux et Mémoires* 10 (1987) 159-206, esp. 195-6. For assistance in tracking down the German version I would like to thank Dr Jochen Twele for his expertise and persistence.

29 Ananias of Shirak, no. 4: Kokian, *Anania von Schirak*, 114. Herakleios, *Novel* 1, which established the permitted number of clergy attached to the church of St. Sophia: I. and P. Zepos (eds.), *Jus Graecoromanum* (Athens 1931) I, 27-30. J. Konidares, 'Die Novellen des Kaisers Herakleios' in *Fontes Minores* 5 (1982) 33-106.

30 Theophanes Continuatus, IV, 29, ed. I. Bekker (Bonn 1838) 185-90; P. Lemerle, *Byzantine Humanism*, *Byzantina Australiensia* 3 (Canberra 1986) 174-8.

31 Vogel, 'Byzantine Science', 265; see also id., 'Byzanz, ein Mittler – auch in der Mathematik – zwischen Ost und West', in *Beiträge zur Geschichte der Arithmetik* (Munich 1978) 35-53.

32 On Leo's education, see Lemerle, *Humanism*, 171-5. Of course this information is probably pure invention. The source of the ancient texts studied by Leo remains unknown, but on his reputation cf. K. Vogel: 'Without him [Leo] the revival of mathematical studies in the West based on Greek texts is well nigh inconceivable', 'Byzantine Science', 265.

33 According to Theophanes Continuatus, John was actually a relative of Leo, an assertion which may be intended to tar both scholars with the same iconoclast brush.

34 It is D'Orville 301 in the Bodleian Library, Oxford, see E.M. Thompson, *An Introduction to Greek and Latin Palaeography* (Oxford 1912) 221-3 and pl. 53; H. Gamillscheg and D. Harlfinger, *Die Repetorium der griechischen Kopisten*, IA (Vienna 1981), nos. 365, 183.

35 K. Vogel, 'Buchstabenzechnung und indische Ziffern in Byzanz', in *Akten des XI Internationalen Byzantinisten-Kongresses*, eds. F. Dölger and H.-G. Beck (Munich 1960) 660-4.

36 As claimed by Lemerle, *Humanism*, 196, but see now the important note by N.G. Wilson, 'Three Byzantine scribes', *Greek, Roman and Byzantine Studies* 14 (1973) 223, in which he shows that the scholion is by a later hand and may not refer to Leo the Mathematician.

37 Lemerle, *Humanism*, 196.

38 Ibid., 196-7.

39 See above, n. 18, and Heath, *Diophantus*, 15. As Kurt Vogel aptly noted, it is much more likely that a copy of Diophantus would have been made at Leo's instigation (c. 830-70) than in the previous century, see 'Byzanz, ein Mittler', esp. 37-8.

40 Lauxtermann, 'Epigram', has demonstrated the importance of Leo's role in the transmission, 244-8.
41 Ibid. 247-8.
42 Paton, 5. 28.
43 Lemerle, *Humanism*, 171.
44 Hypatia: *Suidae Lexikon*, ed. A. Adler (Leipzig 1935) 4. 644-6. *Quadrivium*: A. Diller, *Isis* 36 (1946) 132.
45 Edited by Tannery, *Diophanti*, II, 78-112; it occurs in his main work, the *Quadrivium*, possibly written between 1285 and 1292, see Allard, 'Les Scholies', 670-2; C. Constantinides, *Higher Education in Byzantium in the Thirteenth and Early Fourteenth Centuries* (Nicosia 1982) 62-3.
46 Wilson, *Scholars of Byzantium*, 241 (an autograph is preserved in the Vatican library).
47 Tannery, II, 125-260.
48 Ibid. 232-3.
49 On the editorial work of Planudes and his efforts to obtain additional copies of Diophantus, see Allard, 'Les Scholies', 669-71; Constantinides, *Higher Education*, 71-4; R. Browning, 'Recentiores non deteriores', *Bulletin of the Institute of Classical Studies* 11 (1960) 11-21, repr. in his *Studies on Byzantine History, Literature and Education* (London 1977).
50 On these folios, which were discovered by Tannery, see Allard, 'L'*Ambrosianus Et. 157 Sup.*', 219-34.
51 Wilson, *Scholars of Byzantium*, 234; I. Ševčenko, 'Theodore Metochites, the Chora and the intellectual trends of his time', in P. A. Underwood (ed.), *The Kariye Djami*, vol. 4 (Princeton 1975), 19-91, esp. 24, 37, 42, 44. Cf. Constantinides, *Higher Education*, 66-89.
52 R.J. Loenertz, *Demetrius Cydones, Correspondence*, vol. II (Studi e Testi 208: Vatican 1960), 287. I thank Professor Anne Tihon for pointing out the significance of this reference to Diophantus.
53 Cameron, *The Greek Anthology*, 75-7, 317-20, 345-53.
54 Wilson, *Scholars of Byzantium*, 235-6.
55 Kazhdan, *Oxford Dictionary of Byzantium*, I, 431-2.
56 Tannery, *Diophanti*, II, 260, noted that the scholion was made by a second hand; Wilson, *Scholars of Byzantium*, 233 and esp. addenda 279.
57 His collection consisted of 482 Greek manuscripts and 264 in Latin, see L. Labowsky, *Bessarion's Library and the Biblioteca Marciana* (Rome 1979).
58 See Morse, 'Reception', 190-1; Allard, 'Les Scholies', 665, dates this manuscript to the thirteenth century, but most authorities assign it to the early fifteenth century.
59 Morse, 'Reception', 61-2; Heath, *Diophantus*, 20.
60 Heath, ibid.
61 See Allard, 'Les Scholies', 668.
62 Vaticanus graecus 191, acquired from the collection of thirty Greek MSS previously owned by Cardinal Isidore of Kiev, after his death in 1463 (see R. Devreesse, *Le fond de la Bibliothèque Vaticane des origines à Paul V* (Vatican 1965), 42; and Vaticanus graecus 200, a copy of Bessarion's Venice manuscript, which is recorded in the inventory of 1481, *ibid*, 93. Manuscripts of Diophantus were frequently lent by the Vatican; Vaticanus graecus 304 in 1522, Vaticanus graecus 191 in 1518, 1522 and 1531; the Marciana library also lent its copy in 1545-6: see P. L. Rose, *The Italian Renaissance of Mathematics* (Geneva 1975) 38, 46.
63 See J. Cassinet, 'The first arithmetic book of Francisco Maurolico … a step towards a theory of numbers', in C. Hay (ed.), *Mathematics from Manuscript to Print, 1300-1600* (Oxford 1988) 162-80.
64 Heath, *Diophantus*, 21-2; Morse, 'Reception', ch. iii.
65 Heath, *Diophantus*, 16; Morse, 'Reception', 193; M. Gardthausen and K. Vogel, *Die*

griechischen Schreiber des Mittelalters und der Renaissance (Leipzig 1909, 196) 181-4, lists over 50 manuscripts made by Giovanni. See now the corrected list in E. Gamillscheg and D. Harlfinger (eds.), *Repertorium der griechischen Kopisten, 800-1600*, II, A Frankenreich (Vienna 1989) 99-100; and esp. B. Rainò, *Giovanni Onorio da Maglie. Trascrittore di codici greci* (Bari 1972).

66 On this important manuscript, see H. Omont, *Inventaire sommaire des manuscrits grecs de la Bibliothèque nationale* (4 vols., Paris 1886-93) II, 249. On the contribution of Viète, see G. J. Whitrow, 'Why did mathematics begin to take off in the sixteenth century?', in Hay, *Mathematics*, 264-70, esp. 269-70.

67 Morse, 'Reception', 193-4.

68 Heath, *Diophantus*, 22-6; Morse, 'Reception', 194-212.

69 Heath, *Diophantus*, 26-8.

70 Heath, *Diophantus*, 28-9; Singh, *Fermat*, 64-6.

71 Heath, *Diophantus*, 144-5.

72 Singh, *Fermat*, 79-98.

73 See n. 3 above; Wiles has received $50,000, all that is left of the original prize money.

74 Singh, *Fermat*, 60, repeats the claim that 1453 was 'the vital turning point for Western mathematics', but ignores both the immense contribution of medieval Arab scholars to the development of the science and, even more crucially for Diophantus, the editorial investment and pedagogic use of his writings by generations of Byzantines. N. G. Wilson, *From Byzantium to Italy. Greek Studies in the Renaissance* (London 1992) 162, rightly stresses the damage caused by the sack of 1204 and emphasizes that late Byzantine scholars conserved all the ancient texts that survived this much greater disaster. For a more balanced view of the impact of Greek manuscripts in the West after 1453, see L. Jardine, *Worldly Goods* (London 1996) 57-64.

75 Singh, *Fermat*, 61-2; cf. Morse, Reception, 259-60, quoting Bachet's *Pleasant and Delectable Problems which are Solved by Means of Numbers* (Lyon 1612), which she characterizes as 'almost the first in the genre of recreational mathematics'.